Mass Spectrum and the Nature of Neutrinos.

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Taking as input the best fit solar neutrino anomaly description, MSW LMA, and the tritium beta decay results we estimate the allowed range of neutrino masses independently of their nature. Adding the present bound on the effective neutrino mass coming from neutrinoless double beta decay, we narrow this range for Majorana neutrinos. We complete the discussion by considering future perspectives on determining the neutrino masses, when the oscillation data will be improved and the next experiments on $(\beta\beta)_{0\nu}$ and ^3H decay give new bounds or obtain concrete life-times or distortions in the energy distribution.

We know much more about neutrino masses than yet a few years ago. The observed anomalies in atmospheric, solar and possibly the LSND neutrino experiments, which we believe are explained by neutrino oscillations, supplied with the tritium beta decay data give hints on neutrino masses independently of whether they are Dirac or Majorana particles. Additional constraints on Majorana neutrino masses come from the fact that no neutrinoless double beta decay has been observed to this day. In this work we present an up to date analysis and future perspectives of finding the neutrino mass spectrum without any constraints from theoretical models. We consider only the three neutrino case (i.e. without considering the LSND anomaly), and the latest best fit solar neutrino problem solution, the MSW LMA [1]. The oscillation parameters inferred from atmospheric and solar data are given in Table 1. The four neutrino case and other currently acceptable solutions of the solar anomaly are considered elsewhere [3]. As there are definitely two scales of δm^2, $\delta m^2_{atm} \gg \delta m^2_{sol}$, two possible neutrino mass spectra must be considered. The first, known as normal mass hierarchy (A_3) where $\delta m^2_{sol} = \delta m^2_{21} \ll \delta m^2_{32} \approx \delta m^2_{atm}$ and the second, inverse mass hierarchy spectrum (A_3^{inv}) with $\delta m^2_{sol} = \delta m^2_{21} \ll \delta m^2_{atm} \approx -\delta m^2_{31}$. Both schemes are not distinguishable by present experiments. There is hope that future neutrino factories will do that [4].

Two elements of the first row of the mixing matrix $|U_{e1}|$ and $|U_{e2}|$ can be expressed by the

Table 1: The allowed range (95% of CL) and the best fit values of $\sin^2 2\theta$ and δm^2 for the atmospheric neutrino oscillation and the best fit MSW LMA solution of the solar neutrino problem.

	Allowed range		Best fit	
	$\delta m^2[\,\mathrm{eV}^2]$	$\sin^2 2\theta_{solar}$	$\delta m^2[\,\mathrm{eV}^2]$	$\sin^2 2\theta_{solar}$
Atmospheric neutrinos[2]	$(1.5-6)\times 10^{-3}$	$0.84-1$	3.5×10^{-3}	1
Solar neutrinos (MSW LMA)[1]	$(1.5-10)\times 10^{-5}$	$0.3-0.92$	8×10^{-5}	0.66

third element $|U_{e3}|$ and the $\sin^2 2\theta_{solar}$

$$|U_{e1}|^2 = (1-|U_{e3}|^2)\frac{1}{2}(1+\sqrt{1-\sin^2 2\theta_{solar}}), \tag{1}$$

and

$$|U_{e2}|^2 = (1-|U_{e3}|^2)\frac{1}{2}(1-\sqrt{1-\sin^2 2\theta_{solar}}). \tag{2}$$

The value of the third element $|U_{e3}|$ is not fixed yet and only different bounds exist for it. We will take the bound directly inferred from the CHOOZ and SK experiments [5]

$$|U_{e3}|^2 < 0.04 \quad (\text{with } 95\% \text{ of CL}). \tag{3}$$

Since in both schemes there is

$$(m_\nu)^2_{max} = (m_\nu)^2_{min} + \delta m^2_{solar} + \delta m^2_{atm}, \tag{4}$$

the oscillation experiments alone give

$$(m_\nu)_{max} \geq \sqrt{\delta m^2_{solar} + \delta m^2_{atm}}, \tag{5}$$

and

$$|m_i - m_j| \leq \sqrt{\delta m^2_{solar} + \delta m^2_{atm}}. \tag{6}$$

Translating the above into numbers (again at 95% CL) [2] we end up with

$$(m_\nu)_{max} \geq 0.04 \text{ eV}, \quad |m_i - m_j| < 0.08 \text{ eV}. \tag{7}$$

The next important data comes from the tritium beta decay experiments. The following bound has been lately obtained [6]

$$\left[\sum_{i=1}^{3}|U_{ei}|^2 m_i^2\right]^{1/2} \equiv m_\beta < \kappa' = 2.2 \text{ eV} \tag{8}$$

this obviously leads only to the double inequality

$$(m_\nu)_{min} \leq m_\beta \leq (m_\nu)_{max}. \tag{9}$$

Therefore

$$0 \leq (m_\nu)_{min} \leq 2.2 \text{ eV}. \tag{10}$$

$(m_\nu)_{max}$ remains unfortunately unlimited from above. Supplying the tritium decay with oscillations we find that [7]

$$m_\beta^2 = (m_\nu)^2_{min} + \Omega_{scheme}, \tag{11}$$

and

$$(m_\nu)^2_{max} = m_\beta^2 + \Lambda_{scheme}, \tag{12}$$

where Ω and Λ are scheme dependent. For example, in the A_3 scheme

$$\Omega(A_3) = (1 - |U_{e1}|^2)\delta m^2_{solar} + |U_{e3}|^2 \delta m^2_{atm}, \tag{13}$$

and

$$\Lambda(A_3) = |U_{e1}|^2 \delta m^2_{solar} + (1 - |U_{e3}|^2)\delta m^2_{atm}. \tag{14}$$

This provides limits for both $(m_\nu)_{min}$ and $(m_\nu)_{max}$

$$0 \leq (m_\nu)_{min} \leq \sqrt{(\kappa')^2 - \Omega^{min}_{scheme}}, \tag{15}$$

and

$$\sqrt{\delta m^2_{solar} + \delta m^2_{atm}} \leq (m_\nu)_{max} \leq \sqrt{(\kappa')^2 + \Lambda^{max}_{scheme}}, \tag{16}$$

where this time Ω^{min}_{scheme} and Λ^{max}_{scheme} are the allowed minimal and maximal values. With the present bound on m_β (Eq. 8) we recover practically the same range for $(m_\nu)_{min}$ from Eq. 10, but for $(m_\nu)_{max}$ we obtain from Eq. 16

$$0.04 \text{ eV} \leq (m_\nu)_{max} \leq 2.2 \text{ eV}. \tag{17}$$

With the help of Eq. 11 we plot the range of m_β values for a given $(m_\nu)_{min}$ in Fig. 1 and **??** for the A_3 and A_3^{inv} schemes respectively. We see that the knowledge of m_β determines satisfactorily $(m_\nu)_{min}$ for $m_\beta > 0.04$ eV(0.2 eV) in the $A_3(A_3^{inv})$ case. Within this range of m_β values the spectrum of neutrino masses can be determined independently of the neutrino nature (Dirac or Majorana), since none of the above depends on it. This would be the only possible way to find the masses if the neutrinos were Dirac particles. In future the value of m_β should go down to 0.5 eV[8]. If a value in this range is confirmed, then the spectrum is determined. If not, however, lower values of m_β will require investigation, although this seems to be exteremely difficult.

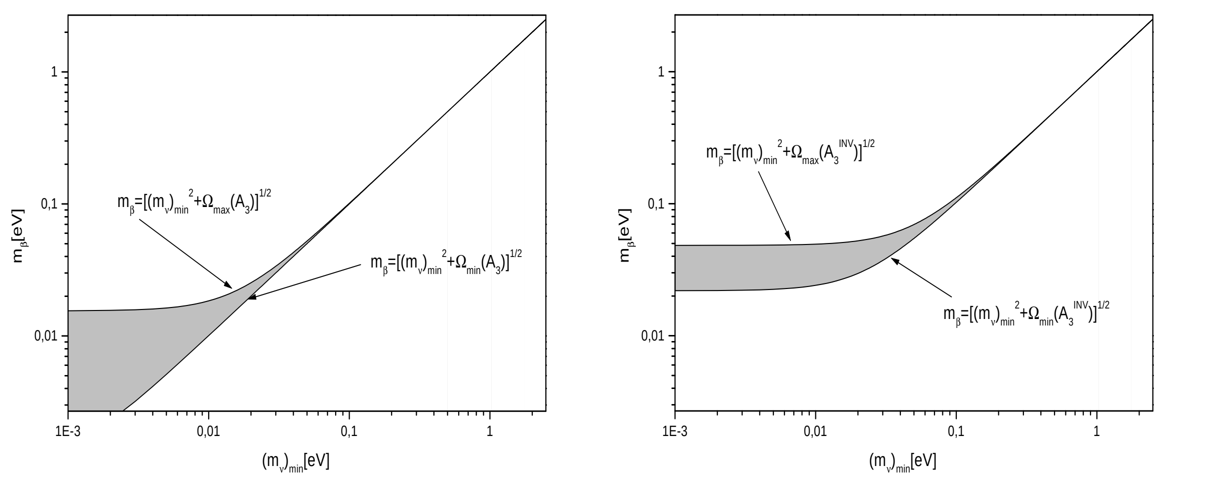

Figure 1: The allowed range of m_β values for a given $(m_\nu)_{min}$ in the A_3 scheme (left) and the A_3^{inv} scheme (right).

For Majorana neutrinos there is one additional constraint, namely the following combination of neutrino masses and mixing matrix elements can be determined from the neutrinoless double beta decay of nuclei[9]

$$\langle m_\nu \rangle = \sum_{i=1}^{3} U^2_{ei} m_i. \tag{18}$$

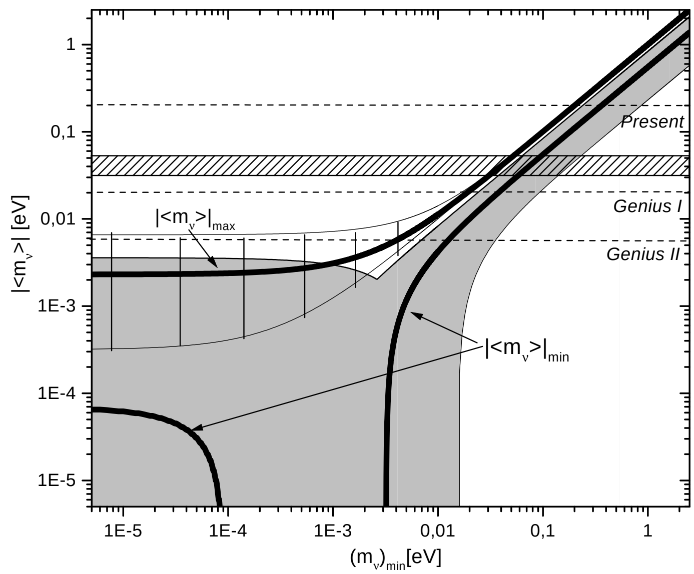

Figure 2: Lower $|\langle m_\nu \rangle|_{min}$ and upper $|\langle m_\nu \rangle|_{max}$ limits of the range of $|\langle m_\nu \rangle|$ as function of $(m_\nu)_{min}$ in the case of the A_3 scheme and the best fit values of the oscillation parameters (solid lines). The shaded and hashed regions represent the smearing of the limits if the present error bars of the oscillation data are taken into account. The present and future (GENIUS I and II) bounds on $|\langle m_\nu \rangle|$ are featured. The horizontal band is an example of a GENIUS I positive result with a possible error. $|\langle m_\nu \rangle| \in (0.02 - 0.05)$ eV.

The present experiments give only a bound, as no such decay has been observed [10]

$$|\langle m_\nu \rangle| < 0.2 \text{ eV}. \tag{19}$$

There are future plans to go down to $|\langle m_\nu \rangle| \simeq 0.02$ eV or even to $|\langle m_\nu \rangle| \simeq 0.006$ eV [11]. Do we have a chance of finding the Majorana mass spectrum if a value of $|\langle m_\nu \rangle|$ is found within such a small range [12]? This answer as we will see is not very promising. We shall neglect the difficulties connected with the determination of $|\langle m_\nu \rangle|$ from the half life time of germanium [13]. As the phases of U_{ei} remain unknown, we are not in position to predict the value of $|\langle m_\nu \rangle|$. However, the lower $|\langle m_\nu \rangle|_{min}$ and upper $|\langle m_\nu \rangle|_{max}$ ranges as function of $(m_\nu)_{min}$ can be inferred [14]. They are shown in Fig. 2 for the A_3 scheme and for the MSW LMA solar neutrino problem solution. The shaded and hashed regions give the uncertainties connected with the allowed ranges of the input parameters ($\sin^2 2\theta_{solar}$, δm^2_{atm} (Table 1) and $|U_{e3}|^2$ (Eq. 3). Future better knowledge of these parameters will reduce the uncertainty regions shown in Fig. 2, but the min-max range caused by the unknown CP phases will remain.

The present experimental bound on $|\langle m_\nu \rangle|$ (Eq. 19) gives the following limit on the possible $(m_\nu)_{min}$ for Majorana neutrinos

$$(m_\nu)_{min} < 0.86 \text{ eV}. \tag{20}$$

This bound strongly depends on the unknown oscillation parameters, most notably on $\sin^2 2\theta_{solar}$. In Fig. 3 we plot this dependence for two different sets of δm^2_{atm} and $|U_{e3}|^2$ values. The limit given in Eq. 20 is valid for $\sin^2 2\theta_{solar} = 0.92$, $|U_{e3}|^2 = 0.04$ and $\delta m^2_{atm} = 6 \times 10^{-3}$ eV^2. If in future, the $(\beta\beta)_{0\nu}$ experiments observe no decay, and a new bound is only found, the next better limit that can be derived from Fig. 3 (with the present oscillation results), is

$$(m_\nu)_{min} < 0.092 \text{ eV} \quad \text{GENIUS I}, \tag{21}$$

and

$$(m_\nu)_{min} < 0.037 \text{ eV} \quad \text{GENIUS II}. \tag{22}$$

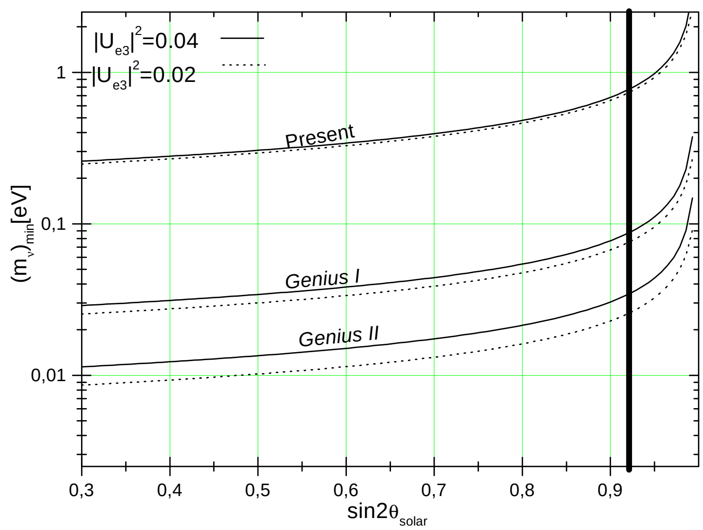

Figure 3: The dependence of the bound on $(m_\nu)_{min}$ on $\sin^2 2\theta_{solar}$ for two different sets of δm^2_{atm} and $|U_{e3}|^2$. The different lines represent the values of $|\langle m_\nu \rangle|$ for different experiments.

In the contrary situation, where a value $|\langle m_\nu \rangle|_{min} \in (0.2 - 0.006)$ eV is confirmed, we can try to predict the Majorana neutrino mass spectrum. The result depends on the value of $|\langle m_\nu \rangle|$ and on the precision of the oscillation parameters. In Fig. 2 a possible band of $|\langle m_\nu \rangle|$ values is given with the GENIUS project estimates. The band crosses the region of values allowed by oscillations giving the possible values of $(m_\nu)_{min}$

$$(m_\nu)^{min(\beta\beta)_{0\nu}}_{min} \leq (m_\nu)_{min} \leq (m_\nu)^{max(\beta\beta)_{0\nu}}_{min}. \tag{23}$$

With the present day uncertainties on the oscillation parameters, the range of possible values determined by Eq. 23 is not satisfactorily small. For example, with $|\langle m_\nu \rangle| \simeq 0.05$ eV

$$(m_\nu)_{min} \in (0.03 - 0.6) \text{ eV}. \tag{24}$$

For smaller values of $|\langle m_\nu \rangle|$ we can only say that $(m_\nu)_{min} < 0.2$ eV. A better knowledge of the oscillation parameters changes the situation slightly. For example, if the oscillation parameters are known with negligible error bars for $|\langle m_\nu \rangle| \simeq 0.05$ eV, then the range Eq. 24 changes to

$$(m_\nu)_{min} \in (0.04 - 0.1) \text{ eV}. \tag{25}$$

The ignorance of the CP breaking phases in the mixing matrix is fully responsible for this smearing.

The bounds on the effective neutrino mass $|\langle m_\nu \rangle|$ in the inverse hierarchy mass scheme A_3^{inv} and the MSW LMA solution of the solar neutrino problem are depicted in Fig. 4. We see that the present bound on $|\langle m_\nu \rangle|$ (Eq. 19), gives a similar limit on the possible range of $(m_\nu)_{min}$ of Majorana neutrino masses

$$(m_\nu)_{min} < 0.86 \text{ eV}. \tag{26}$$

The first stage of GENIUS can yield

$$(m_\nu)_{min} < 0.077 \text{ eV}, \tag{27}$$

while the second would exclude the A_3^{inv} scheme.

In conclusion, the present data allow for the following statements

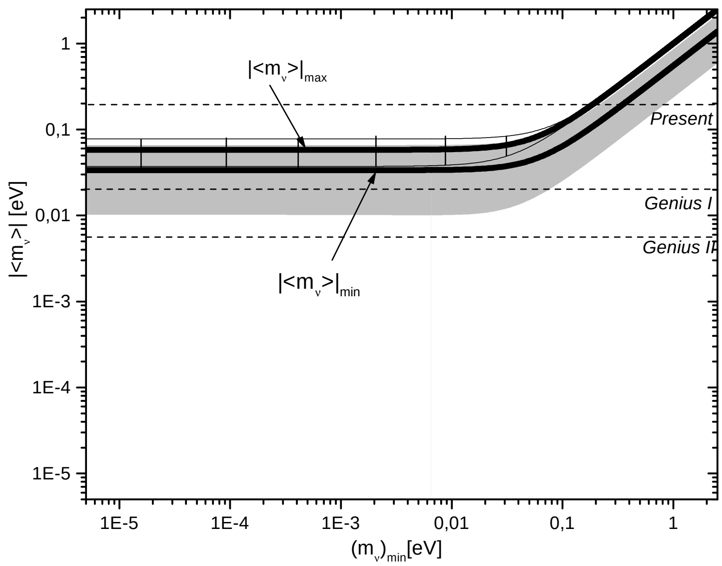

Figure 4: Lower $|\langle m_\nu\rangle|_{min}$ and upper $|\langle m_\nu\rangle|_{max}$ limits of the range of $|\langle m_\nu\rangle|$ as function of $(m_\nu)_{min}$ in the case of the A_3^{inv} scheme and the best fit values of the oscillation parameters (solid lines). The shaded and hashed regions represent the smearing of the limits if the present error bars of the oscillation data are taken into account. The present and future (GENIUS I and II) bounds on $|\langle m_\nu\rangle|$ are featured.

- we are not able to distinguish between Dirac and Majorana neutrinos.
- the allowed range of masses for Dirac neutrinos is wider than for Majorana, but the latter depends strongly on the oscillation parameters.
- the oscillation and tritium beta decay experiments are able to determine the spectrum of neutrino masses for values of m_β which differ in the A_3 ($m_\beta \geq 0.04$ eV) and the A_3^{inv} ($m_\beta \geq 0.2$ eV) schemes.
- the oscillation and $(\beta\beta)_{0\nu}$ experiments are able to find the range of possible $(m_\nu)_{min}$ values. However, this range is not small even with oscillation parameters of negligible error bars.

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